ON THE EFFECT OF HORIZONTAL DEFORMATION ON INSAR SUBSIDENCE ESTIMATES

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ABSTRACT

One of the limitations of InSAR is that it is only capable of measuring a 3D projection of a real deformation vector on the radar line of sight. Therefore it is not possible to retrieve the full displacement vector from a single InSAR measurement. The optimal solution for this limitation is a combination of InSAR measurements from different imaging geometries. However, this solution is not applicable in areas where InSAR data are available in one particular geometry only. In this paper, a new approach will be discussed for 3D decomposition of InSAR deformation measurements for the case of subsidence. We use the hypothesis of a physical relation between horizontal displacement and vertical deformation. Using this hypothesis, we propose an iterative strategy for 3D decomposition of InSAR subsidence measurements. The proposed approach is tested on the PSI results of a complex subsidence field in Friesland, the Netherlands. Finally, the results are validated by comparison with available leveling data.

Key words: Persistent scatterer ; InSAR; Horizontal deformation; Subsidence.

1. INTRODUCTION

In the last 20 years, InSAR and different timeseries InSAR approaches have shown a great capability in mapping the earth surface deformation induced by various geophysical mechanisms. One of the shortcomings of the technique is that InSAR observations are only sensitive to surface deformation away from or towards the satellite. In other words, InSAR can only measure a projection of the real 3D deformation vector on the LOS of the satellite. Wright et al. (2004) showed that by using InSAR observations with at least three different imaging geometries it is possible to overcome this limitation and to decompose the InSAR LOS measurements into north, east, and up components. In the cases in which InSAR data with only two different geometries are available, two of the three components can be retrieved. The so called speckle-tracking method (Derauw, 1999; Fialko et al., 2005) can be also used to derive the deformation in the azimuth look direction. However, this method is only effective for very large deformations and coherent areas.

When observations of only one particular imaging geometry are available, InSAR measurements need to be interpreted with care. Ignoring the effect of a potential horizontal component assuming InSAR is more sensitive to vertical deformation can introduce large errors in the final deformation estimates. Generally for the decomposition of LOS measurements with only one imaging geometry, assumptions are necessary on the characteristics of the deformation signal. In this paper we use the hypothesis that the horizontal displacement is proportional to the tilt (i.e., first spatial derivative of vertical deformation)(Kratzsch, 1983). Using this hypothesis, we propose a method to derive the horizontal components of a subsidence field using InSAR data of only a single imaging geometry. We tested the approach on the subsidence field in Friesland, the Netherlands. The Persistent Scatterer Interferometry (PSI) results of this area are used and the final decomposition are validated using the available leveling data.

2. EFFECT OF HORIZONTAL DEFORMATION ON INSAR MEASUREMENTS

The InSAR measurement $d_{LOS}$ is the projection of a 3D displacement vector $d$ with components $d_n$, $d_e$, and $d_u$, in North, East, and UP direction, respectively, into the LOS direction. Assuming an incidence angle $\theta_{inc}$ and a satellite orbit with heading (azimuth) of $\alpha_h$, we have (Hanssen, 2001):

$$d_{LOS} = d_u \cos(\theta_{inc}) - d_{ALD} \sin(\theta_{inc}), \quad (1)$$

where $d_{ALD}$ is the projection of the multiple horizontal components on the azimuth look direction (ALD):
Considering near polar orbits and the incidence angle of current SAR satellites (\(20 - 30^\circ\)), InSAR observations are mostly sensitive to the vertical component while their sensitivity to the north component is minimal. For example, for an incidence angle of \(\theta_{inc} \approx 23^\circ\) and a heading of \(\alpha_h \approx 200^\circ\), a sensitivity decomposition of LOS deformation is \([0.92, -0.13, 0.37]^{\top} [d_u, d_n, d_e]^{\top}\). Nevertheless, the fact that InSAR observations are mostly sensitive to vertical deformation should not mistakenly result in ignorance of the horizontal component in the deformation analyses. Converting a LOS measurement to vertical while ignoring its horizontal component results in an error \(\Delta d_u\) in the up component which can be computed as:

\[
\Delta d_u = \tan(\theta_{inc}) \cdot \frac{[d_e \cos(\alpha_h) - d_n \sin(\alpha_h)]}{d_{ALD}}.
\]

Practically, this means that assuming \(\theta_{inc} = 23^\circ\), the maximum absolute error in \(d_u\) can reach \(\tan(\theta_{inc})\), or 42\% of the horizontal component (i.e., when the horizontal motion is parallel to the ALD), and the minimum error is zero when the horizontal component is perpendicular to the ALD (\(d_{ALD} = 0\)). In order to clarify this effect, we simulated a simple Gaussian subsidence bowl with centripetal horizontal displacement (Figures 1A and 1B). The maximum vertical deformation is 30 mm in the center, and the maximum absolute horizontal displacement is 8 mm. If this simulated subsidence bowl is observed with an incident angle of 23\(^\circ\) and a heading of 190\(^\circ\), the error introduced by converting LOS observations to vertical while ignoring the horizontal component is visualized in Figure 1C. It shows that by ignoring the horizontal component, we overestimate the up component in areas whose horizontal displacement is towards the satellite (i.e., blue area in Figure 1C) and we underestimate the up component in areas with horizontal displacements away from the satellite.

3. **SUBSIDENCE, HORIZONTAL DISPLACEMENT, AND TILT**

First we define the three main subsidence parameters used in this study. These parameters are illustrated in Figure 2.

1. **Vertical Subsidence** \(d_z\): refers to vertical movement of a point at the earth’s surface.
2. **Horizontal displacement** \(d_h\): is the horizontal surface motion induced by subsidence. Horizontal displacement is zero at the deepest point and at the limits of a subsidence bowl. It reaches its maximum in areas with maximum tilt.

Based on similarities between horizontal displacement curve and tilt, Kratzsch (1983) made the hypothesis that the horizontal displacement is proportional to the tilt:

\[
d_h = K d'_r = K \frac{\partial}{\partial r}(d_z),
\]

where \(K\) is an unknown proportionally factor. The \(K\) value is dependent to the geophysical/geological characteristics of the overburden. Kratzsch (1983) defined the rule of thumb in order to approximate \(K\) as:

\[
K = 0.33 R,
\]

where \(R\) is the radius of the subsidence area. Note that the exact value of \(K\) is unknown.

4. **3D DECOMPOSITION OF LOS OBSERVATIONS**

In this section we introduce an algorithm for 3D decomposition of InSAR LOS measurements using the hypothesis of a linear relation between tilt and horizontal displacement. As tilt was defined as the first derivative of the
4.1. Computation of tilt

The Computation of tilt is performed in two subsequent steps:

1. **Interpolation:** In order to derive a spatial derivative, we prefer a spatially continuous field. Here we use ordinary Kriging (interpolation) to build the continuous subsidence field from sparsely sampled data. Assume for each data point we have the X and Y coordinate and the vertical deformation. First, a regularly sampled grid is defined. Then, for each grid point \((x, y)\) the vertical deformation is interpolated by Kriging. The result is a continuous subsidence field \(S(x, y)\).

2. **Differentiate:** After interpolation, the first spatial derivative of the interpolated subsidence field \(S(x, y)\) in X and Y direction is computed by 2D convolution of a \(X\) and \(Y\) gradient kernel and the subsidence field. The gradient kernels in X and Y direction are:

   \[
   g_x = \frac{1}{8 \times \text{gridsize}} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}, \quad (7)
   \]

   \[
   g_y = \frac{1}{8 \times \text{gridsize}} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}. \quad (8)
   \]

The first spatial derivative (tilt) of the subsidence field \(S\) in X and Y direction is defined as:

\[
S'_x(x, y) = \frac{d}{dx}S(x, y) = g_x \ast S(x, y), \quad (9)
\]

and

\[
S'_y(x, y) = \frac{d}{dy}S(x, y) = g_y \ast S(x, y), \quad (10)
\]

where \(\ast\) is the convolution operator. Then, the absolute value of the tilt \(S'\) will be:

\[
S'(x, y) = \left[ (S'_x(x, y))^2 + (S'_y(x, y))^2 \right]^{1/2}, \quad (11)
\]

and the direction of the tilt vector at each grid point \((x, y)\) is:

\[
\alpha(x, y) = \arctan\left( \frac{S'_y(x, y)}{S'_x(x, y)} \right). \quad (12)
\]

4.2. 3D Decomposition

Let \(d_{\text{LOS}}\) be the vector of LOS observations at data points with coordinates \(X\) and \(Y\). Assuming a known proportionally factor \(K\), we propose the following steps for the 3D decomposition of InSAR observations.

- **Step1:** Initialization: set \(d_n = 0\) and \(d_e = 0\).
- **Step2:** convert \(d_{\text{LOS}}\) to the vertical component \(d_u\) assuming the horizontal displacements \(d_n\) and \(d_e\).
- **Step3:** compute the subsidence field \(S\) and its first derivatives in X and Y direction \((S'_x\) and \(S'_y)) using the method described in the section 4.1.
- **Step4:** compute the horizontal displacement field in X and Y direction \((S_x\) and \(S_y)) using equation:

\[
S_x = KS'_x, S_y = KS'_y. \quad (13)
\]

- **Step5:** Interpolate \(S_x\) and \(S_y\) on each data point \((X, Y)\) by kriging to estimate \(d_n\) and \(d_e\).
- **Step6:** Iterate steps 2-5 until \(d_u\) converges.

It needs to be emphasized that the value of \(K\) is crucial in this estimation, see section 5.3.

5. CASE STUDY: SUBSIDENCE IN THE NORTH FRIESLAND, THE NETHERLANDS

5.1. Introduction of the study area

In Friesland, one of the northern provinces in the Netherlands, gas production commenced in 1988 west of the city of Franeker (Figure 3). In 1995, solution mining started production just north-west of the gas field. In
2003, an additional salt mining well north of the gas field (BAS3) became operational. Both gas and salt extraction in the area have resulted in subsidence of the ground level, which has been monitored extensively using periodic leveling campaigns. Figure 3 shows the study area and the cumulative subsidence field between 1988 and 2006 estimated from leveling data (Oranjewoud, 2007).

Figure 3. Top: Friesland subsidence area: the black dash line shows the boundary of the gas reservoir; HRLs are the locations of the gas wells, and BASs are locations of the salt caverns; Bottom: cumulative subsidence (in mm) between 1988 and 2006 derived from leveling data (Oranjewoud, 2007).

5.2. PSI processing

In this study, Persistent Scatterer Interferometry (PSI) results of Friesland were used. Using ERS1/2 as well as Envisat data, the full period between 1992 to 2008 was analyzed. Main difficulties in performing PSI in this area are the rural characteristics of the earth surface, relatively high deformation rates, and non-linear deformation especially in the areas where the subsidence of the two deformation mechanisms interferes with each other. The Delft PSI (DePSI) approach (van Leijen et al., 2006; Kampes, 2005) was used. To test our 3D decomposition approach, we selected the cumulative subsidence for the periods 1998-2000 and 2003-2006. These cumulative measurements were derived from the deformation time-series produced by PSI processing. The reason for selecting these two specific time intervals is that they correspond to the time of leveling campaigns which makes comparison possible. Figures 4A and 4B show the cumulative deformation for the two selected time intervals for the detected persistent scatterers.

Figure 4. PSI results: A) cumulative deformation between 1998 and 2000 (based on the ERS1/2 data). B) cumulative deformation between 2003 and 2006 (based on the Envisat data).

5.3. Estimation of $K$ factor

Generally, there are three ways to derive $K$. First, one can use the rule of thumb eq. (6) assuming a known radius for the subsidence area; Second, it is possible to use auxiliary observations, for examples in areas where both leveling (vertical) and GPS (horizontal) measurements are available, to derive the tilt from leveling data, and estimate $K$. Third, one can use InSAR data with different imaging geometries (e.g., ascending and descending tracks) in time intervals at which these data are available, decompose the deformation in horizontal and vertical, derive the tilt from vertical deformation estimates, and finally estimate $K$.

As the third approach is based on empirical data, and relies least on assumptions, we apply this approach here.
We use the PSI results of the period 1998-2000 for which both ascending and descending ERS data are available. Nearby PS with distance shorter than 50 m are assumed to have the same deformation and are grouped as one cluster. For each cluster the LOS deformations are decomposed to the vertical component and the horizontal component along descending look direction (see APPENDIX 1). The tilt is computed along the descending look direction based on the method described in section 4.1. Using kriging, the decomposed horizontal components are interpolated on the same grid as the tilt. Figures 5A and 5B show the tilt and the interpolated horizontal component. The factor $K$ was estimated using least squares on eq. (5). In the estimation, only grid points in which there are PS are used. Figure 5C shows tilt values versus horizontal displacements and the estimated $K$ of 997.82. By reverse engineering of Kratzsch’s rule, see (6), the value of $K$ should follow from a subsidence bowl with a radius of 3 km, which is a reasonable value. Although this $K$ factor was estimated based on the tilt component in ALS, we can use it in other directions assuming a homogeneous and isotropic medium.

5.4. 3D decomposition

Assuming no significant change in reservoir parameters, we can use the $K$ factor we estimated for the 1998-2000 period for other time intervals as well. Cumulative subsidence of periods 1998-2000 and 2003-2006 are interpolated by kriging on the same grid and added together. From this subsidence field (over 5 years), the tilt was computed. Figures 6A and 6B show the interpolated subsidence field and its corresponding tilt respectively. Using the iterative approach of section 4.2, we decompose this subsidence field to up, north, and east components (Figures 7A, 7B, and 7C).

5.5. Comparison with leveling

In order to validate our approach, we compared the derived vertical component with leveling. First, the leveling deformations of periods 1998-2000 and 2003-2006 were added together and interpolated on the same grid which was used for 3D decomposition. This interpolated subsidence field derived from leveling is referred to as $S_{level}$. Let $S_{H=0}$ be the subsidence field derived by converting LOS radar observations to the vertical assuming no horizontal displacement. And let $S_v$ be the vertical subsidence field derived by our 3D decomposition approach. First we look at the difference between $S_{level}$ and $S_{H=0}$. The comparison was only made for grid points in which both PSI and leveling data are available. We expect that the difference between $S_{level}$ and $S_{H=0}$ may be
Figure 7. Decomposed components: A) up component, B) north component, and C) east component. Arrows show the direction of the horizontal components.

partially due to our wrong assumption about no horizontal deformations and therefore to be correlated with the tilt component in ALD as InSAR is only sensitive to the horizontal component in ALD. Figures 8A, 8B show the map of the differences and the tilt in ALD respectively. Figure 8C shows the correlation between the two maps (normalized) in locations where both leveling and PS are available. We found the correlation coefficient to be not higher than 0.77 which confirms our expectation. Apparently, the difference between leveling and PSI can be partly explained by the effect of the horizontal displacement.

After 3D decomposition, the vertical component of the radar result, \( S_v \), can be compared with the leveling results \( S_{level} \); see the difference between the two in Figure 9. Comparing with 8A, we can see that generally the differences with leveling were reduced after decomposition. For example west of Franeker (the red circle in Figure 9) we can clearly see this reduction. However in some areas like the area west of BAS3 (the black box in Figure 9) the differences increased. The main reason is that the number of PS is very limited east of BAS3, so we obtained erroneous interpolation and therefore a wrong estimate of derivative (tilt) in this area. Subsequently, this leads to a wrong estimation of the horizontal component. The main conclusion is that a crucial requirement for our approach is having a good spatial sampling of PS. In the areas with limited or zero number PS the computed tilt can be over/under estimated leading to a wrong 3D decomposition of deformation vector. Finally, Figure 10 shows the histograms and standard deviation of the differences after 3D decomposition and also for the case in which we ignored the horizontal component. We can observe the reduction in the standard deviation of the differences from 11 mm to 8 mm which indicates the generally good performance of the proposed method.

Figure 8. A) difference between \( S_{level} \) and \( S_{H=0} \), B) the tilt component in ALD, and C) correlation between the normalized tilt and the normalized differences between \( S_{level} \) and \( S_{H=0} \).

6. CONCLUSIONS

We showed that ignoring the horizontal components of a subsidence field can result in an over/under estimation of the estimated vertical deformation. The results of the Friesland case study showed that the difference between leveling and InSAR can be partly explained by the effect of ignoring the horizontal deformation component. We introduced a method for the 3D decomposition of InSAR line of sight observations in the case of subsidence based on (Kratzsch, 1983). With this method, it is possible to estimate and correct for the horizontal component on the bases of only one track (i.e., one imaging geometry) using the hypothesis of a linear relation between tilt and the horizontal component. The main advantage of this approach is that it only uses data and it is independent of any particular geometrical or physical subsidence model. With this approach, full 3D decomposition of the LOS deformation measurements is feasible. However, a spatial interpolation is necessary for computation of a spatial derivative of subsidence (i.e., tilt). In the case of sparsely sampled data (e.g., PSI results) the performance of the method requires good spatial density of data points in order to get a reliable interpolation and therefore a reliable tilt. In the case of conventional InSAR when continuous subsidence field is available, our method can be used directly without any spatial interpolation.

Further study will focus on the validity of the linear relation between tilt and horizontal components on different reservoirs, quality assessment of 3D decomposed components, dependency of the \( K \) factor and geophysical reservoir parameters, more optimal spatial interpolation for
subsidence monitoring in order to overcome the limitation in areas with a low PS spatial density, and finally the influence of 3D decomposition of LOS deformations on geophysical reservoir modeling.

ACKNOWLEDGMENTS

We would like to thank to European Space Agency (ESA) for their support via the Cat-3538 project and the data supply for this research.

REFERENCES


7. APPENDIX 1: DECOMPOSITION OF LOS DEFORMATION

For a cluster of two PS with ascending and descending imaging geometries, a decomposition to the vertical component and one horizontal component in one particular direction is feasible. A system of equations for the decomposition in the vertical component and the horizontal component in the descending look direction is:

$$
\begin{bmatrix}
d_{\text{asc}}^{\text{los}} \\
d_{\text{desc}}^{\text{los}}
\end{bmatrix} = A \cdot
\begin{bmatrix}
d_{\text{up}} \\
d_{\text{hald}}
\end{bmatrix},
$$

(14)

where

$$
A = \begin{bmatrix}
\cos \theta^{\text{asc}} & \sin \theta^{\text{asc}} / \cos \Delta \alpha \\
\cos \theta^{\text{desc}} & \sin \theta^{\text{desc}}
\end{bmatrix},
$$

(15)

with:

- $d_{\text{los}}$ deformation along LOS
- $d_{\text{up}}$ vertical deformation
- $d_{\text{hald}}$ projection of horizontal deformation in descending azimuth look direction
- $\theta$ incident angle
- $\Delta \alpha$ satellite heading difference between ascending and descending mode.